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# Asynchronous domain decomposition methods for continuous casting problem

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## Abstract

Two asynchronous domain decomposition methods (which appear to be a two-stage Schwarz alternating algorithms) to solve the finite difference schemes approximating dynamic continuous casting problem are theoretically and numerically studied. Fully implicit and semi-implicit (implicit for the diffusion operator while explicit for the nonlinear convective term) finite difference schemes are considered. Unique solvability of the finite difference schemes as well as a monotone dependence of the solution on the right-hand side (the so-called comparison theorem) are proved. Geometric rate of convergence for the iterative methods is investigated, the comparison theorem being the main tool of this study. Numerical results are included and analyzed.

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**Keywords:** Continuous casting problem; Finite difference scheme; Domain decomposition; Finite-dimensional inclusion; Asynchronous iteration; Parallel solution

## 1. Introduction

The general idea of the Schwarz alternating methods is to solve the boundary-value problem restricted to each subdomain, using as the boundary conditions the function values of the approximate solution of the neighboring subdomains. One of the advantages of the additive Schwarz is that the solutions in the subdomains can be handled by different processors of a parallel computer. However, due to the mutual waits among the processors when a synchronous method is applied, it leads to high loss of calculating time. To exploit the asynchronous parallel computing capacity of a

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